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# WORKSHEET

# Inverse Trigonometric Functions

## Unsolved with Given Solutions

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## Inverse Trigonometric Functions

**Q1: Evaluate $\sin^{-1}(\sqrt{3}/2)$.**

**Q2: Simplify $\cos^{-1}(-1/2) + \sin^{-1}(1/2)$.**

**Q3: Solve for x: $\tan^{-1}(x) = \pi/6$.**

**Q4: Find the derivative of $y = \cos^{-1}(3x)$.**

**Q5: Evaluate $\tan^{-1}(1) + \tan^{-1}(1/3)$.**

**Q6: Solve the equation: $2\sin^{-1}(x) = \pi/3$.**

**Q7: Find the domain of $f(x) = \tan^{-1}(2x+1)$.**

**Q8: Evaluate $\sin^{-1}(\sin(5\pi/4))$.**

**Q9: Simplify $\tan^{-1}(1/\sqrt{3}) + \cot^{-1}(\sqrt{3})$.**

**Q10: Find the integral of $1/\sqrt{(1-x^2)}$ dx from 0 to 1/2.**

**Q11: Solve for x: $\cos^{-1}(x) + \sin^{-1}(x) = \pi/2$.**

**Q12: Evaluate $\cot^{-1}(-1) - \tan^{-1}(1)$.**

**Q13: Find the range of $f(x) = \sin^{-1}(2x-1)$.**

**Q14: Solve the equation: $\tan^{-1}(x) + \tan^{-1}(2x) = \pi/4$.**

**Q15: Evaluate $\sin^{-1}(\cos(7\pi/6))$.**

**Q16: Evaluate $\cos^{-1}(-\sqrt{2}/2)$.**

**Q17: Simplify $\sin^{-1}(x) + \sin^{-1}(\sqrt{(1-x^2)})$ for $0 \leq x \leq 1$.**

**Q18: Solve for x: $2\tan^{-1}(x) = \pi/3$.**

**Q19: Find the derivative of $y = \tan^{-1}(x^2)$.**

**Q20: Evaluate $\sin^{-1}(1/2) + \cos^{-1}(1/2)$.**

# Solutions

## Inverse Trigonometric Functions

**Solution 1:**

To evaluate $\sin^{-1}(\sqrt{3}/2)$, we need to recall the common angle values in trigonometry.

$\sin 30° = 1/2$

$\sin 60° = \sqrt{3}/2$

Since $\sin^{-1}$ is the inverse function of sin, we have:

$\sin^{-1}(\sqrt{3}/2) = 60°$

Converting to radians:

$60° \times (\pi/180°) = \pi/3$ radians

Therefore, **$\sin^{-1}(\sqrt{3}/2) = \pi/3$ radians or 60°.**

**Solution 2:**

Let's approach this step-by-step:

First, recall that $\cos^{-1}(-1/2) = 2\pi/3$ radians or 120°.

This is because $\cos 120° = -1/2$.

Next, $\sin^{-1}(1/2) = \pi/6$ radians or 30°.

This is because $\sin 30° = 1/2$.

Now, we can add these values:

$\cos^{-1}(-1/2) + \sin^{-1}(1/2) = 2\pi/3 + \pi/6$

$= 4\pi/6 + \pi/6$

$= 5\pi/6$ radians or 150°

Therefore, **$\cos^{-1}(-1/2) + \sin^{-1}(1/2) = 5\pi/6$ radians or 150°.**

**Solution 3:**

To solve this equation, we need to apply the tangent function to both sides:

$\tan(\tan^{-1}(x)) = \tan(\pi/6)$

The tan and $\tan^{-1}$ on the left side cancel out, leaving:

$x = \tan(\pi/6)$

We know that $\tan(\pi/6) = 1/\sqrt{3}$

Therefore, $x = 1/\sqrt{3}$.

**Solution 4:**

To find the derivative of $y = \cos^{-1}(3x)$, we can use the chain rule and the derivative formula for arccosine:

The derivative of $\cos^{-1}(x)$ is $-1/\sqrt{(1-x^2)}$.

Let $u = 3x$, then $y = \cos^{-1}(u)$

$dy/dx = dy/du \times du/dx$

$= (-1/\sqrt{(1-u^2)}) \times 3$

$= -3/\sqrt{(1-(3x)^2)}$

$= -3/\sqrt{(1-9x^2)}$

Therefore, **the derivative of $y = \cos^{-1}(3x)$ is $dy/dx = -3/\sqrt{(1-9x^2)}$.**

**Solution 5:**

Let's approach this step-by-step:

$\tan^{-1}(1) = \pi/4$ radians or 45°

To find $\tan^{-1}(1/3)$, we can use the formula:

$\tan^{-1}(1/3) = \pi/2 - \tan^{-1}(3) = \pi/2 - \pi/3 = \pi/6$

Now, we can add these values:

$\tan^{-1}(1) + \tan^{-1}(1/3) = \pi/4 + \pi/6$

$= 3\pi/12 + 2\pi/12$

$= 5\pi/12$ radians or 75°

Therefore, **$\tan^{-1}(1) + \tan^{-1}(1/3) = 5\pi/12$ radians or 75°.**

**Solution 6:**

Let's solve this step-by-step:

First, divide both sides by 2:

$\sin^{-1}(x) = \pi/6$

Now, apply the sine function to both sides:

$\sin(\sin^{-1}(x)) = \sin(\pi/6)$

The sin and $\sin^{-1}$ on the left side cancel out:

$x = \sin(\pi/6)$

We know that $\sin(\pi/6) = 1/2$

Therefore, **$x = 1/2$ is the solution to the equation $2\sin^{-1}(x) = \pi/3$.**

### Solution 7:

To find the domain of $f(x) = \tan^{-1}(2x+1)$, we need to consider the domain of the arctangent function.

The arctangent function ($\tan^{-1}$) is defined for all real numbers.

Therefore, 2x+1 can be any real number.

We can express this as an inequality:

$-\infty < 2x+1 < \infty$

Solving for x:

$-\infty < 2x+1 < \infty$

$-\infty - 1 < 2x < \infty - 1$

$-\infty < 2x < \infty$

$-\infty < x < \infty$

Therefore, **the domain of $f(x) = \tan^{-1}(2x+1)$ is all real numbers, or $(-\infty, \infty)$.**

### Solution 8:

To evaluate this expression, let's think about the range of $\sin^{-1}$ and the value of $\sin(5\pi/4)$:

The range of $\sin^{-1}$ is $[-\pi/2, \pi/2]$.

$\sin(5\pi/4) = -\sqrt{2}/2$

$\sin^{-1}(\sin(5\pi/4)) = \sin^{-1}(-\sqrt{2}/2)$

We know that $\sin(-\pi/4) = -\sqrt{2}/2$

Since $-\pi/4$ is within the range of $\sin^{-1}$, we have:

$\sin^{-1}(-\sqrt{2}/2) = -\pi/4$

Therefore, **$\sin^{-1}(\sin(5\pi/4)) = -\pi/4$ radians or -45°.**

**Solution 9:**

Let's approach this step-by-step:

First, recall that $\tan^{-1}(1/\sqrt{3}) = \pi/6$ radians or 30°.

Next, note that $\cot^{-1}(x) = \tan^{-1}(1/x)$.

So, $\cot^{-1}(\sqrt{3}) = \tan^{-1}(1/\sqrt{3}) = \pi/6$ radians or 30°.

Now, we can add these values:

$\tan^{-1}(1/\sqrt{3}) + \cot^{-1}(\sqrt{3}) = \pi/6 + \pi/6 = \pi/3$ radians or 60°

Therefore, **$\tan^{-1}(1/\sqrt{3}) + \cot^{-1}(\sqrt{3}) = \pi/3$ radians or 60°.**

**Solution 10:**

This integral is the definition of the arcsine function. Let's solve it step-by-step:

$\int 1/\sqrt{(1-x^2)}\, dx = \sin^{-1}(x) + C$

Now, we need to evaluate this from 0 to 1/2:

$[\sin^{-1}(x)]_0^{1/2} = \sin^{-1}(1/2) - \sin^{-1}(0)$

We know that:

$\sin^{-1}(1/2) = \pi/6$

$\sin^{-1}(0) = 0$

Therefore:

$[\sin^{-1}(x)]_0^{1/2} = \pi/6 - 0 = \pi/6$

Thus, **the integral of $1/\sqrt{(1-x^2)}\, dx$ from 0 to 1/2 is $\pi/6$.**

**Solution 11:**

Let $y = \cos^{-1}(x)$. Then $\cos y = x$.

From the Pythagorean identity, we know that $\sin^2 y + \cos^2 y = 1$.

Substituting $\cos y = x$, we get:

$\sin^2 y + x^2 = 1$

Solving for sin y:

$\sin y = \sqrt{(1-x^2)}$

Now, $\sin^{-1}(\sin y) = y = \cos^{-1}(x)$

Therefore, $\sin^{-1}(\sqrt{(1-x^2)}) = \cos^{-1}(x)$

This means that for any x in the domain of both functions:

$\cos^{-1}(x) + \sin^{-1}(x) = \cos^{-1}(x) + \sin^{-1}(\sqrt{(1-x^2)}) = \pi/2$

Therefore, **this equation is true for all x in the interval [-1, 1].**

**Solution 12:**

Let's approach this step-by-step:

First, $\cot^{-1}(-1) = 3\pi/4$ radians or 135°.

This is because cot 135° = -1.

Next, $\tan^{-1}(1) = \pi/4$ radians or 45°.

This is because tan 45° = 1.

Now, we can subtract:

$\cot^{-1}(-1) - \tan^{-1}(1) = 3\pi/4 - \pi/4 = \pi/2$ radians or 90°

Therefore, **$\cot^{-1}(-1) - \tan^{-1}(1) = \pi/2$ radians or 90°.**

**Solution 13:**

The domain of $\sin^{-1}$ is [-1, 1]. So:

$-1 \leq 2x-1 \leq 1$

Solving these inequalities:

$0 \leq 2x \leq 2$

$0 \leq x \leq 1$

When $x = 0$, $2x-1 = -1$, so $\sin^{-1}(2x-1) = -\pi/2$

When $x = 1$, $2x-1 = 1$, so $\sin^{-1}(2x-1) = \pi/2$

Since $\sin^{-1}$ is a continuous and increasing function, its range will be all values between these extremes.

Therefore, **the range of $f(x) = \sin^{-1}(2x-1)$ is $[-\pi/2, \pi/2]$.**

**Solution 14:**

Recall the formula for tan(A+B): $\tan(A+B) = (\tan A + \tan B) / (1 - \tan A \tan B)$

In our case, $\tan(\pi/4) = \tan(\tan^{-1}(x) + \tan^{-1}(2x))$

We know that $\tan(\pi/4) = 1$, so:

$1 = (x + 2x) / (1 - x(2x))$

$1 = 3x / (1 - 2x^2)$

Cross multiply:

$1 - 2x^2 = 3x$

Rearrange:

$2x^2 + 3x - 1 = 0$

This is a quadratic equation. We can solve it using the quadratic formula:

$x = [-3 \pm \sqrt{(9+8)}] / 4 = (-3 \pm \sqrt{17}) / 4$

The positive solution satisfies the original equation:

$x = (-3 + \sqrt{17}) / 4 \approx 0.5616$

Therefore, **the solution to $\tan^{-1}(x) + \tan^{-1}(2x) = \pi/4$ is**

**$x = (-3 + \sqrt{17}) / 4$.**

**Solution 15:**

First, let's evaluate $\cos(7\pi/6)$:

$\cos(7\pi/6) = \cos(\pi + \pi/6) = -\cos(\pi/6) = -\sqrt{3}/2$

Now we have:

$\sin^{-1}(-\sqrt{3}/2)$

Recall that $\sin^{-1}$ is the inverse function of sin in the range $[-\pi/2, \pi/2]$.

We know that $\sin(-\pi/3) = -\sqrt{3}/2$

Since $-\pi/3$ is within the range of $\sin^{-1}$, we have:

$\sin^{-1}(-\sqrt{3}/2) = -\pi/3$

Therefore, **$\sin^{-1}(\cos(7\pi/6)) = -\pi/3$ radians or -60°.**

**Solution 16:**

To evaluate $\cos^{-1}(-\sqrt{2}/2)$, let's recall some common angle values:

$\cos(3\pi/4) = -\sqrt{2}/2$

The range of $\cos^{-1}$ is $[0, \pi]$, and $3\pi/4$ is within this range.

Since $\cos^{-1}$ is the inverse function of cos, we have:

$\cos^{-1}(-\sqrt{2}/2) = 3\pi/4$

Therefore, **$\cos^{-1}(-\sqrt{2}/2) = 3\pi/4$ radians or 135°.**

**Solution 17:**

Let $\theta = \sin^{-1}(x)$. Then $\sin \theta = x$.

From the Pythagorean identity, we know that $\sin^2 \theta + \cos^2 \theta = 1$.

Substituting $\sin \theta = x$, we get:

$x^2 + \cos^2 \theta = 1$

Solving for $\cos \theta$:

$\cos \theta = \sqrt{(1-x^2)}$

Now, $\cos^{-1}(\cos \theta) = \theta = \sin^{-1}(x)$

Therefore, $\cos^{-1}(\sqrt{(1-x^2)}) = \sin^{-1}(x)$

This means that for any x in the interval [0, 1]:

$\sin^{-1}(x) + \sin^{-1}(\sqrt{(1-x^2)}) = \sin^{-1}(x) + \cos^{-1}(\sqrt{(1-x^2)}) = \pi/2$

Hence, **$\sin^{-1}(x) + \sin^{-1}(\sqrt{(1-x^2)})$ simplifies to $\pi/2$ for $0 \leq x \leq 1$.**

**Solution 18:**

First, divide both sides by 2:

$\tan^{-1}(x) = \pi/6$

Now, apply the tangent function to both sides:

$\tan(\tan^{-1}(x)) = \tan(\pi/6)$

The tan and $\tan^{-1}$ on the left side cancel out:

$x = \tan(\pi/6)$

We know that $\tan(\pi/6) = 1/\sqrt{3}$

Therefore, **$x = 1/\sqrt{3}$ is the solution to the equation $2\tan^{-1}(x) = \pi/3$.**

**Solution 19:**

To find the derivative of $y = \tan^{-1}(x^2)$, we can use the chain rule and the derivative formula for arctangent:

The derivative of $\tan^{-1}(x)$ is $1/(1+x^2)$.

Let $u = x^2$, then $y = \tan^{-1}(u)$

Using the chain rule:

$dy/dx = dy/du \times du/dx$

$= [1/(1+u^2)] \times [2x]$

$= 1/(1+(x^2)^2) \times 2x$

$= 2x/(1+x^4)$

Therefore, **the derivative of $y = \tan^{-1}(x^2)$ is $dy/dx = 2x/(1+x^4)$.**

**Solution 20:**

First, $\sin^{-1}(1/2) = \pi/6$ radians or 30°.

This is because sin 30° = 1/2.

Next, $\cos^{-1}(1/2) = \pi/3$ radians or 60°.

This is because cos 60° = 1/2.

Now, we can add these values:

$\sin^{-1}(1/2) + \cos^{-1}(1/2) = \pi/6 + \pi/3$

$= \pi/2$ radians or 90°

Therefore, **$\sin^{-1}(1/2) + \cos^{-1}(1/2) = \pi/2$ radians or 90°.**